NEWTON'S LAW OF UNIVERSAL GRAVITATION

Note that planets move in nearly circular orbits around the sun. As mentioned earlier, the force that keeps these planets from coasting off in a straight line is a **gravitational force.** The gravitational force is a field force that always exists between two masses, regardless of the medium that separates them. It exists not just between large masses like the sun, Earth, and moon but between any two masses, regardless of size or composition. For instance, desks in a classroom have a mutual attraction because of gravitational force. The force between the desks, however, is small relative to the force between the moon and Earth because the gravitational force is proportional to the product of the objects' masses.

gravitational force

the mutual force of attraction between particles of matter

Gravitational force acts such that objects are always attracted to one another. Examine the illustration of Earth and the moon in **Figure 7-12.** Note that the gravitational force between Earth and the moon is attractive, and recall that Newton's third law states that the force exerted on Earth by the moon, $\mathbf{F}_{mE}$, is equal in magnitude to and in the opposite direction of the force exerted on the moon by Earth, $\mathbf{F}_{Em}$.

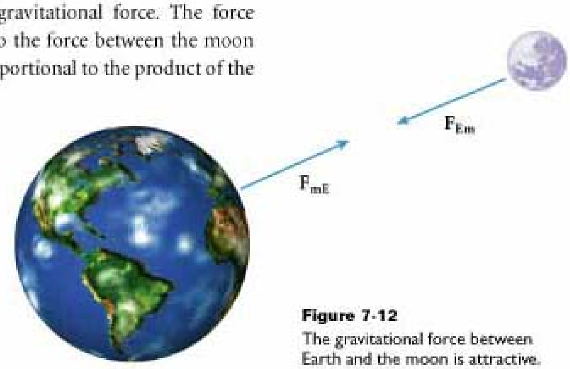

Figure 7-12
The gravitational force between Earth and the moon is attractive. According to Newton's third law, $F_{Em} = F_{mE}$.

Gravitational force depends on the distance between two masses

If masses m_1 and m_2 are separated by distance r, the magnitude of the gravitational force is given by the following equation:

NEWTON'S LAW OF UNIVERSAL GRAVITATION

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$\text{gravitational force} = \text{constant} \times \frac{\text{mass 1} \times \text{mass 2}}{(\text{distance between center of masses})^2}$$

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G is a universal constant called the *constant of universal gravitation;* it can be used to calculate gravitational forces between any two particles and has been determined experimentally.

$$G = 6.673 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$$

The law of universal gravitation is an example of an *inverse-square law,* because the force varies as the inverse square of the separation. That is, the force between two masses decreases as the masses move farther apart.

Did you know?

The astronomer Johannes Kepler proposed that planets orbit the sun in elliptical paths. However, some scientists continued to believe that Earth was the center of the solar system until Sir Isaac Newton showed that elliptical orbits could be predicted using his laws of motion.

Gravitational force is localized to the center of a spherical mass

The gravitational force exerted by a spherical mass on a particle outside the sphere is the same as it would be if the entire mass of the sphere were concentrated at its center. For example, the force on an object of mass m at Earth's surface has the following magnitude:

$$F_g = G\frac{M_E m}{R_E^2}$$

M_E is Earth's mass and R_E is its radius. This force is directed toward the center of Earth. Note that this force is in fact the weight of the mass, mg.

$$mg = G\frac{M_E m}{R_E^2}$$

By substituting the actual values for the mass and radius of Earth, we can find the value for g and compare it with the value of free-fall acceleration used throughout this book.

Because m occurs on both sides of the equation above, these masses cancel.

$$g = G\frac{M_E}{R_E^2} = \left(6.673 \times 10^{-11}\,\frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\frac{5.98 \times 10^{24}\ \text{kg}}{(6.37 \times 10^6\ \text{m})^2} = 9.83\ \text{m/s}^2$$

This value for g is approximately equal to the value used throughout this book. The difference is due to rounding the values for Earth's mass and radius.

SAMPLE PROBLEM 7I

Gravitational force

PROBLEM

Find the distance between a 0.300 kg billiard ball and a 0.400 kg billiard ball if the magnitude of the gravitational force is 8.92×10^{-11} N.

SOLUTION

Given: $m_1 = 0.300$ kg $\quad m_2 = 0.400$ kg $\quad F_g = 8.92 \times 10^{-11}$ N

Unknown: $r = ?$

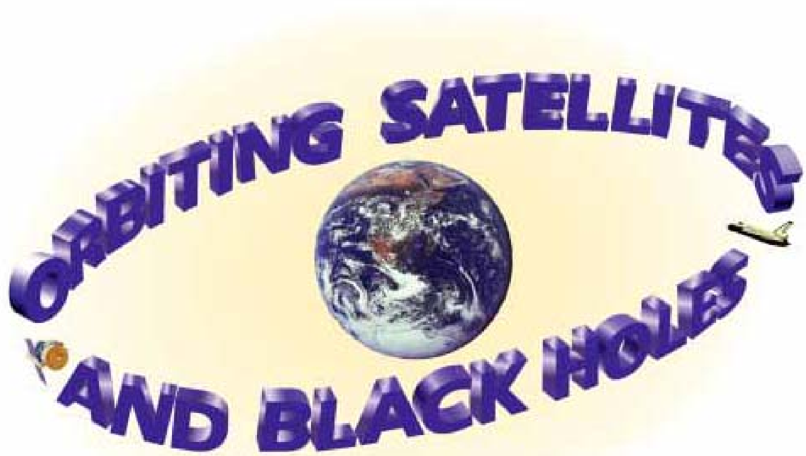
Use the equation for Newton's Law of Universal Gravitation.

$$r^2 = \frac{G}{F_g} m_1 m_2 = \frac{6.673 \times 10^{-11}\frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}}{} (0.300\ \text{kg})(0.400\ \text{kg})$$

$$= 8.97 \times 10^{-2}\ \text{m}^2$$

$$\boxed{r = \sqrt{8.97 \times 10^{-2}\ \text{m}^2} = 3.00 \times 10^{-1}\ \text{m}}$$

PHYSICS ON THE EDGE



Projectiles and satellites

As explained in Chapter 3, when a ball is thrown parallel to the ground, the motion of the ball has two components of motion: a horizontal velocity, which remains unchanged, and a vertical acceleration, which equals free-fall acceleration. Given this analysis, it may seem confusing to think of the moon and other satellites in orbit around Earth as projectiles. But in fact, satellites are projectiles. As shown in **Figure 7-13,** the larger the velocity parallel to Earth's surface, the farther the projectile moves before striking Earth. Note, however, that at some large velocity, the projectile returns to its point of origin without moving closer to Earth. In this case, the gravitational force between the projectile and Earth is just great enough to keep the projectile from moving along its inertial straight-line path. This is how satellites stay in orbit.

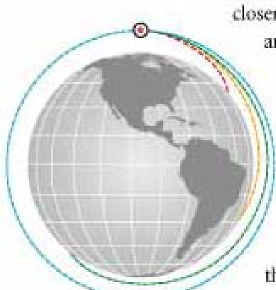


Figure 7-13
When the speed of a projectile is large enough, the projectile orbits Earth as a satellite.

Escape speed

When the speed of an object, such as a rocket, is greater than the speed required to keep it in orbit, the object can escape the gravitational pull of Earth and soar off into space. The object soars off into space when its initial speed moves it out of the range in which the gravitational force is significant. Mathematically, the value of this *escape speed* (v_{esc}) is given by the following equation:

$$v_{esc} = \sqrt{\frac{2MG}{R}}$$

Earth's radius, R, is about 6.37×10^6 m, and its mass is approximately 5.98×10^{24} kg. Thus, the escape speed of a projectile from Earth is 1.12×10^4 m/s. (Note that this value does not depend on the mass of the projectile in question.)

As the mass of a planet or other body increases and its radius decreases, the escape speed necessary for a projectile to escape the gravitational pull of that body increases, as shown in **Figure 7-14.** If the body has a very large mass and a small radius, the speed necessary for a projectile to escape the gravitational pull of that body reaches very high values. For example, an object with a mass three times that of the sun but with a diameter of about 10 km would require an escape speed equal to the speed of light. In other words, the force of gravity such an object exerts on a projectile is so great that even light does not move fast enough to escape it.

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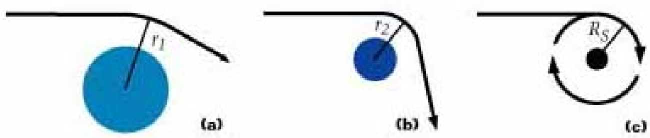

Figure 7-14
(a) A projectile that passes near any massive object will be bent from its trajectory. **(b)** The more compact the object, the nearer the projectile can approach and the greater the escape speed it needs. **(c)** If the object is so small and massive that a projectile's escape speed at distance R_S is greater than the speed of light, then the object is classified as a black hole.

Black holes

The existence of such massive objects was first predicted in 1916 by Karl Schwarzschild, who used his solutions to Einstein's general-relativity equations to predict their properties. Thus, the distance from the center of the object to the circular orbit at which escape speed is equal to the speed of light is called the *Schwarzschild radius*. Because light cannot escape from any point within the sphere defined by the Schwarzschild radius, no information can be obtained about events that occur in this region. Hence, the edge of the sphere is called the *event horizon*, and the apparently lightless region within it is called a *black hole*.

Recent observations have provided strong evidence for the existence of black holes. Tremendous amounts of X rays and other radiation have been observed coming from regions that are near visible stars, although no stars appear to be the source of the radiation. If the visible star has a black hole as a companion, it could be losing some of its outer atmosphere to the black hole, and those atmospheric gases could be emitting radiation as they accelerate closer to the black hole. Candidates for black holes of these types include Scorpius X-1 and Cygnus X-1.

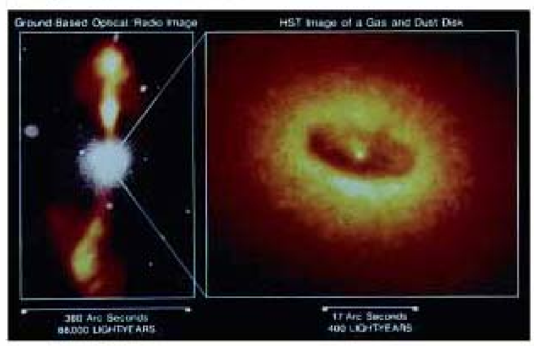

Figure 7-15
The gas jets and the central disk shown at right, created by combining an optical image with a radio telescope image of NGC 4261, show signs of a black hole in this galaxy's center.

The large amount of energy that galactic centers produce suggests to many astrophysicists that the energy sources are supermassive black holes. The galaxy NGC 4261, shown in **Figure 7-15,** is likely to have a black hole at its center. Some astronomers believe the Milky Way, our own galaxy, contains a black hole about the size of our solar system.

Physics Problem Set 12-b Fall 2008

Name: ___________________________ Date Assigned: 11/13/2008
Date Due: 11/18/2008

Gravitational force

1. If the mass of each ball in Sample Problem 7I is 0.800 kg, at what distance between the balls will the gravitational force between the balls have the same magnitude as that in Sample Problem 7I? How does the change in mass affect the magnitude of the gravitational force?
2. Mars has a mass of about 6.4×10^{23} kg, and its moon Phobos has a mass of about 9.6×10^{15} kg. If the magnitude of the gravitational force between the two bodies is 4.6×10^{15} N, how far apart are Mars and Phobos?
3. Find the magnitude of the gravitational force a 67.5 kg person would experience while standing on the surface of each of the following planets:

Planet	m	r
a. Earth	5.98×10^{24} kg	6.37×10^{6} m
b. Mars	6.34×10^{23} kg	3.43×10^{6} m
c. Pluto	5×10^{23} kg	4×10^{5} m

Answers

1.

2.

3

4. Calculate the escape velocity for the Earth. $v_{esc} = \sqrt{\frac{2m_E G}{r_E}}$, given that m_E = 6E24 kg and r_E = 6.4E6 m. Convert the answer to miles per hour given 1 hr = 3600 s, 1.61 km = 1 mi, and 1 km = 1,000 m.

5. Suppose the Earth became a black hole, i.e. having its mass collapse to a small spherical ball.

A. If the speed-of-light, c = 3.0E8 m/s, what would the Earth's radius have to be so that light could not escape? What is the name of such a radius?

B. Calculate the collapsed Earth's new volume given $V = 1.33\pi r^3$

C. What would the collapsed Earth's density be?

D. What can or cannot happen inside the "event horizon?"

E. Since black hole cannot be seen, i.e. light cannot escape, what evidence to astronomers seek for their exisitence?