

## Group Activity 1: Earth and Solar System

Name(s) \_\_\_\_\_, \_\_\_\_\_  
\_\_\_\_\_, \_\_\_\_\_

**Theory:** Earth's gravitational force is often modeled as though the Earth were an inert sphere of uniform density. Such a body would produce a field of uniform magnitude and direction at all points on its surface. In reality there are slight deviations in both the magnitude and direction of gravity across the surface of the Earth because none of those qualities are exactly true of Earth.

Furthermore, the net force exerted on an object due to the Earth, called apparent gravity or effective gravity varies due to the presence of other forces. A scale or plumb bob measures only this effective gravity. The strength of Earth's apparent gravity varies with latitude, altitude, local topography and geology:

### Part 1: Earth's gravity based on latitude

#### Latitude

Apparent gravity is weaker nearer the equator because the Earth's rotation produces an apparent centrifugal force (inertia).

Gravity provides centripetal force, keeping objects on the surface (and indeed the surface itself) moving in a circular motion. Consider that if the gravity of the Earth were to shut off, objects would fly off into space in the direction of their motion in accordance with Newton's First Law of Motion. Alternatively, if Earth's gravity were weakened so as to match the "centrifugal force" (at, say, the equator where rotational speed is largest) then objects there would appear to float.

At the poles the radius of curvature (how curved something is) is zero, so only this weakened gravity would contribute to weight and objects would not float. In this sense, local gravity (gravity at a particular point on the surface of the Earth) felt as weight is gravity due to the Earth's mass minus the centrifugal force. Because rotational speed decreases as one moves towards the poles, local gravity,  $g$ , increases from  $9.789 \text{ m/s}^2$  at the equator to  $9.832 \text{ m/s}^2$  at the poles.

The second major cause for the difference in gravity at different latitudes is that the Earth's equatorial bulge (itself also caused by centrifugal force/inertia) causes objects at the equator to be farther from the planet's centre than objects at the poles. Because the force due to gravitational attraction between two bodies (the Earth and the object being weighed) varies inversely with the square of the distance between them, objects at the equator experience a weaker gravitational pull than objects at the poles. In combination, the equatorial bulge and the effects of centrifugal force mean that sea-level gravitational acceleration increases from about  $9.780 \text{ m/s}^2$  at the equator to about  $9.832 \text{ m/s}^2$  at the poles, so an object will weigh about 0.5% more at the poles than at the equator.

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**Mathematical models**

If the terrain is at sea level, we can estimate  $g$ :

$$g_{\theta} = 9.780327(1 + 0.0053024\sin^2\theta - 0.0000058\sin^22\theta) \text{ m/s}^2 \text{ (eqt. 1)}$$

Where,

- $g_{\theta}$  = acceleration in  $\text{m/s}^2$  at Latitude:  $\theta$
- This is the International Gravity Formula 1967, the 1967 Geodetic Reference System Formula, Helmert's equation or Clairault's formula

The first correction to this formula is the free air correction (FAC), which accounts for heights above sea level. Gravity decreases with height at a rate which near the surface of the Earth is such that linear extrapolation would give zero gravity at a height of one half the Earth's radius, or about  $9.8 \text{ m/s}^2$  per  $3\,200 \text{ km}$ . Thus:

$$g_{\theta} = [9.780327(1 + 0.0053024\sin^2\theta - 0.0000058\sin^22\theta) - 3.08 \times 10^{-6} h] \text{ m/s}^2 \text{ (eqt. 2)}$$

Where,

- $h$  = height in meters above sea level

**1. You are at Quito, Ecuador ( $0^\circ$  latitude).**

- What is  $g$  to the thousandth's place.
- Calculate  $g$  at increasing latitudes of  $5^\circ$ . Assume sea level. Round to thousandth's place. MS Excel might be helpful or have each member perform 4-5 calculations.
- Plot  $g$  vs.  $\theta$ . Title, label, and use correct units.

When is outer space? It's a good question with several valid answers.

- The Federation Aeronautique Internationale has established the "Karmen Line" at **100 km** (62 mi).
- The United States has designated that astronauts are at **80 km** (50 mi)
- NASA mission control uses **122 km** (76 mi)...when atmospheric drag becomes significant.
- 2009, the University of Calgary used the "Supra-Thermal Ion Imager" to measure the speed and direction of charged particles from space (solar wind, etc). It was determined that **118 km** (73 mi) is the "edge-of-space."

Other significant boundaries (1.61 km = 1 mi):

- Troposphere: **6 – 20 km** (airplanes)
- Stratosphere: **50 km** (weather balloons)
- Mesosphere: **85 km** (non-extinction level meteors usually burn up)
- Thermosphere (lower ionosphere): **85 km – 690 km** (Auroas/Space Shuttle)
- Exosphere (upper ionosphere): **690 km – 10,000 km**

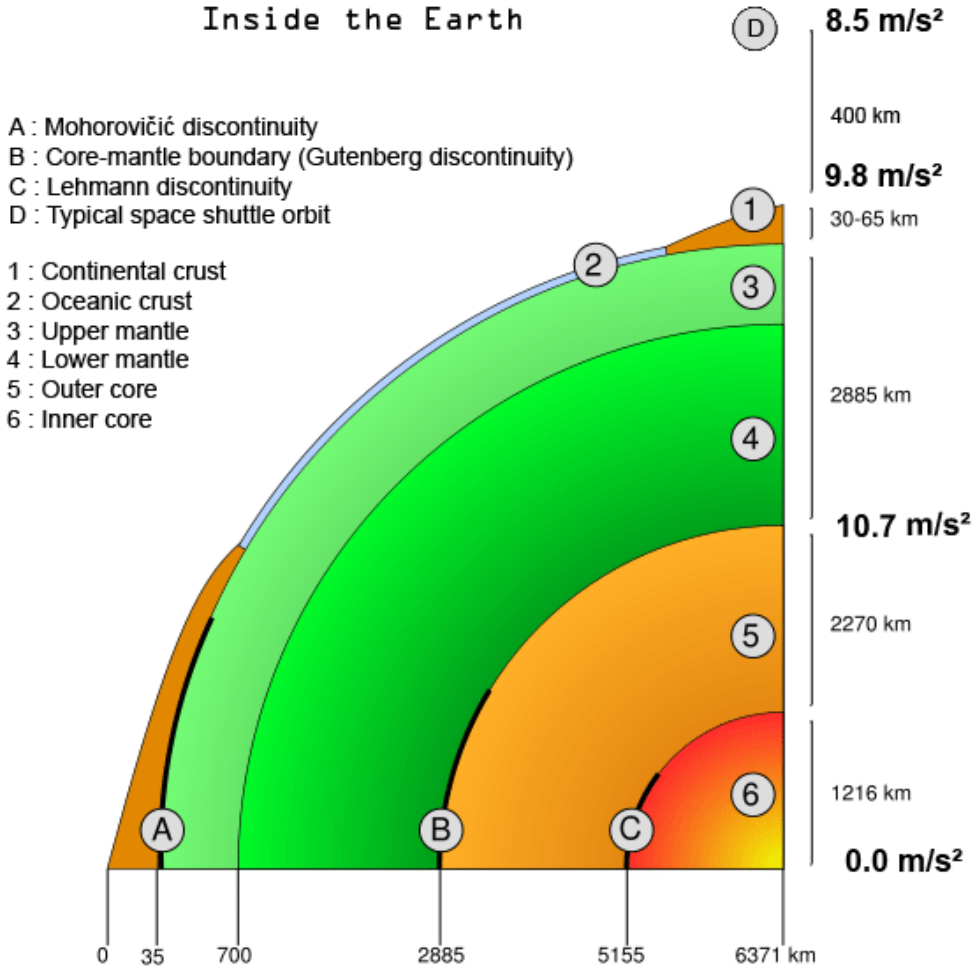
**2. You're still at Quito.**

- calculate  $g$  as function of height for the major boundaries listed above (thousands place).
- Plot  $g$  vs.  $h$  using correct title and labels for the axes.

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- c. Use  $g = G \frac{m_E}{r_E^2}$ ,  $m_E$  = mass of Earth in kg,  $r_E$  = radius of Earth in meters.  
 Plot  $g$  vs.  $r$ , starting with  $r$  at surface of the Earth and increasing at 25,000 m intervals until you reach about 690,000 m. compare with part b.

**Gravitational Field Strength:  
 Inside the Earth**



When calculating “g” in the inside the earth is not simply making r go to zero in

$$g = G \frac{m_E}{r_E^2} \text{ (eqt. 3)}$$

If you could somehow materialize in the interior of the Earth without dying, you would have mass surrounding you. As a consequence, you would experience a net gravitational force based on depth and local geology. To calculate  $g$ , use the equation:

$$g_{inside} = g_{surface} \left[ \frac{r}{r_{Earth}} \right] \text{ (eqt. 4)}$$

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3. Using  $g_{\text{surface}}$  at Quito and eqt. 4, calculate  $g_{\text{in}}$ 
  - a. At  $r = 0$  m and increase at intervals of 200,000 m upto 6,400,000 m
  - b. Plot  $g_{\text{in}}$  vs  $r$ , using correct title, labels, and units.
4. Attach all calculations, Excel tables, and graphs. 1-per group
5. Answer post-lab question on-line, individually.